

$\Sigma_b \rightarrow \Sigma_c$ and $\Omega_b \rightarrow \Omega_c$ weak decays in the light-front quark model

Hong-Wei Ke^{1*}, Xu-Hao Yuan^{2†}, Xue-Qian Li^{3‡}, Zheng-Tao Wei^{3§} and Yan-Xi Zhang^{2¶}

¹ *School of Science, Tianjin University, Tianjin 300072, China*

² *Center for High Energy Physics, Department of Engineering Physics,
Tsinghua University, Beijing 100084, China*

³ *School of Physics, Nankai University, Tianjin 300071, China*

Abstract

The successful operation of LHC provides a great opportunity to study the processes where heavy baryons are involved. In this work we mainly study the weak transitions of $\Sigma_b \rightarrow \Sigma_c$. Assuming the reasonable quark-diquark structure where the two light quarks constitute an axial vector, we calculate the widths of semi-leptonic decay $\Sigma_b \rightarrow \Sigma_c e \nu_e$ and non-leptonic decay modes $\Sigma_b \rightarrow \Sigma_c + M$ (light mesons) in terms of the light front quark model. We first construct the vertex function for the concerned baryons and then deduce the form factors which are related to two Isgur-Wise functions for the $\Sigma_b \rightarrow \Sigma_c$ transition under the heavy quark limit. Our numerical results indicate that $\Gamma(\Sigma_b \rightarrow \Sigma_c e \nu_e)$ is about $1.38 \times 10^{10} \text{s}^{-1}$ and $\Gamma(\Sigma_b \rightarrow \Sigma_c + M)$ is slightly below $1 \times 10^{10} \text{s}^{-1}$ which may be accessed at the LHCb detector. By the flavor SU(3) symmetry we estimate the rates of $\Omega_b \rightarrow \Omega_c$. We suggest to measure weak decays of $\Omega_b \rightarrow \Omega_c$, because Ω_b does not decay via strong interaction, the advantage is obvious.

PACS numbers: 13.30.-a, 12.39.Ki, 14.20.Lq, 14.20.Mr

* khw020056@hotmail.com

† yuanxh@tsinghua.edu.cn

‡ lixq@nankai.edu.cn

§ weizt@nankai.edu.cn

¶ yanxi.zhang@cern.ch

I. INTRODUCTION

In our previous work [1], we investigated the transitions between heavy baryons $\Lambda_b \rightarrow \Lambda_c$ by assuming the baryonic heavy-quark-light-diquark structures in terms of the Light-Front-Quark model (LFQM). The results are reasonably consistent with the available data, so it implies that the whole scenario is realistic in that case, however still needs more studies on its validity in other cases. As noticed that the ground state diquarks in Λ_b and Λ_c are color-anti-triplet scalars. In this work, we continue to consider the transitions of $\Sigma_b \rightarrow \Sigma_c$ because the ground state diquark in $\Sigma_{b(c)}$ is an axial vector. We explore if the difference of the diquark identities would result in distinct behaviors for the transitions and then by comparing with data we are able to gain more insight about the diquark structure.

Thanks to the successful operation of LHC, a remarkable database on baryons, especially on the heavy baryons will be available at LHCb. It enables researchers to closely study the properties of heavy baryons at their production and decay processes.

Since the situation is confronting a radical change, more physicists are turning to concern baryons and look for hints of new physics. For example, as the decay $\Sigma \rightarrow p \mu^+ \mu^-$ was observed[2] the authors of Ref.[3, 4] studied contribution from new physics candidates by analyzing the data. However, as it is well known when one explores possible new physics scenario based on the data, he needs to fully understand the contribution of the standard model (SM) i.e. before attributing the phenomena to new physics a complete analysis on the SM contribution is necessary.

In this work we explore the weak transition of $\Sigma_b \rightarrow \Sigma_c$. The dominant strong decay mode $\Sigma_b \rightarrow \Lambda_b + \pi$ determines the lifetime of Σ_b , thus the weak decays of Σ_b are rare. However, from another aspect, the rare decays of Σ_b may be more sensitive to new physics, so that it is worth a careful study.

Supposing the factorization is valid, the transition between quarks would be fully described by the perturbative theory and calculable, thus the main task for studying $\Sigma_b \rightarrow \Sigma_c$ is to deal with the hadronic transition matrix element. The hadronic matrix elements are determined by non-perturbative QCD and are generally parameterized by some form factors which can be reduced into a few equivalent Isgur-Wise functions under the heavy quark limit[5]. Some authors [6–10] calculated the form factors of the transition $\Sigma_b \rightarrow \Sigma_c$ in various approaches.

The quark-diquark structure that heavy baryons are made of a heavy quark and a light diquark[11–13] is generally considered as a reasonable physics picture for heavy baryons. With the quark-diquark structure the authors of Refs.[1, 6, 7, 14] evaluated the transition rates between heavy baryons and their results are consistent with the available data. It is noted that the diquark stands as a spectator in the transition of $\Sigma_b \rightarrow \Sigma_c$, so that under the heavy quark limit, the spin of light diquark decouples and we may evaluate the rates of the corresponding rare decays in terms of the Isgur-Wise functions. A general analysis suggests that there exist many Isgur-Wise-type functions for a transition between baryons [15] and usually it would be hard to determine them by fitting data. However, with the quark-diquark structure, the number of such functions for the transition $\Sigma_b \rightarrow \Sigma_c$ reduces

into only two.

The light-front quark model (LFQM) is a relativistic quark model which has been applied to study transitions among mesons and the results agree with the data within reasonable error tolerance [16–24], thus we would be tempted to extend its application to calculate the transition of $\Sigma_b \rightarrow \Sigma_c$ as long as the diquark picture is employed. In Ref.[1] we calculated the transition of $\Lambda_b \rightarrow \Lambda_c$ in terms of LFQM. In that work we first constructed the vertex function of $\Lambda_{b(c)}$ and then deduced the form factors for the transition. However the formulas in [1] do not apply to the decay $\Sigma_b \rightarrow \Sigma_c$ because the diquark in $\Lambda_{b(c)}$ is a scalar of color-anti-triplet, but that in $\Sigma_{b(c)}$ is an axial vector as discussed in Refs.[6, 7]. Thus we need to re-construct the vertex function for a $\frac{1}{2}^+$ heavy baryon which is regarded as a bound state of a heavy quark and a light axial vector diquark. Then with the vertex functions of baryons we would derive the transition matrix element which are parametrized by a few form factors, and under the heavy quark limit, we will show that the transition matrix element of $\Sigma_b \rightarrow \Sigma_c$ can be described by two generalized Isgur-Wise functions. Numerically the results obtained in the two approaches are rather close, so it implies that the employed approaches are reasonably consistent with the physical picture.

Since the leptons do not participate in the strong interaction, the semileptonic decay is simple and less contaminated by the non-perturbative QCD effect, therefore study on semileptonic decay might help to test the employed model and/or constrain the model parameters. With the form factors we evaluate the width of the semileptonic decay. Comparing our numerical result with data the model parameters which are hidden in the vertex functions can be fixed. Moreover, the amplitude of the non-leptonic decay $\Sigma_b \rightarrow \Sigma_c + M$ can also be evaluated in a similar way as long as we suppose that the meson current can be factorized out. Moreover, we further investigate the transitions of $\Omega_b \rightarrow \Omega_c$ by assuming the flavor SU(3) symmetry. Since Ω_b does not decay via strong interaction, the weak decays are dominant, so that study on such modes has an obvious advantage.

This paper is organized as follows: after the introduction, in section II we construct the vertex functions of heavy baryons, then derive the form factors for the transition $\Sigma_b \rightarrow \Sigma_c$ in the light-front quark model, then we present our numerical results for the transition $\Sigma_b \rightarrow \Sigma_c$ along with all necessary input parameters in section III, then we also evaluate the transition of $\Omega_b \rightarrow \Omega_c$. Section IV is devoted to our conclusion and discussions.

II. $\Sigma_b \rightarrow \Sigma_c$ IN THE LIGHT-FRONT QUARK MODEL

By the quark-diquark structure[6, 7], the heavy baryon $\Sigma_{b(c)}$ consists of a light 1^+ diquark [ud] and one heavy quark $b(c)$. To insure the quantum number of $\Sigma_{b(c)}$, the orbital angular momentum between the two components is zero, i.e. $l = 0$.

A. the vertex function of $\Sigma_{b(c)}$

In analog to our previous work [1], we construct the vertex function of Σ_Q ($Q = b, c$) where the diquark is an axial vector in the same model. The wavefunction of Σ_Q with total spin $S = 1/2$ and momentum P is

$$\begin{aligned} |\Sigma_Q(P, S, S_z)\rangle &= \int \{d^3\tilde{p}_1\} \{d^3\tilde{p}_2\} 2(2\pi)^3 \delta^3(\tilde{P} - \tilde{p}_1 - \tilde{p}_2) \\ &\times \sum_{\lambda_1} \Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1) C_{\beta\gamma}^\alpha F^{bc} |Q_\alpha(p_1, \lambda_1) [q_{1b}^\beta q_{2c}^\gamma](p_2)\rangle, \end{aligned} \quad (1)$$

with

$$\begin{aligned} \Psi^{SS_z}(\tilde{p}_1, \tilde{p}_2, \lambda_1, \lambda_2) &= \langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle \langle \lambda_2 | \mathcal{R}_M^\dagger(x_2, k_{2\perp}, m_2) | s_2 \rangle \\ &\left\langle \frac{1}{2}s_1; 1s_2 \left| \frac{1}{2}S_z \right. \right\rangle \varphi(x, k_\perp), \end{aligned}$$

where $\langle \frac{1}{2}s_1; 1s_2 | \frac{1}{2}S_z \rangle$ is the C-G coefficients and s_1, s_2 are the spin projections of the constituents (the heavy quark and diquark). A Melosh transformation brings the the matrix elements from the spin-projection-on-fixed-axes representation into the helicity representation and is explicitly written as

$$\langle \lambda_2 | \mathcal{R}_M^\dagger(x_2, k_{2\perp}, m_2) | s_2 \rangle = \xi^*(\lambda_1, m_2) \cdot \xi(s_2, m_2),$$

and

$$\langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle = \frac{\bar{u}(k_1, \lambda_1) u(k_1, s_1)}{2m_1}.$$

Following Refs. [16, 19], the Melosh transformed matrix can be expressed as

$$\begin{aligned} &\langle \lambda_2 | \mathcal{R}_M^\dagger(x_2, k_{2\perp}, m_2) | s_2 \rangle \langle \lambda_1 | \mathcal{R}_M^\dagger(x_1, k_{1\perp}, m_1) | s_1 \rangle \left\langle \frac{1}{2}s_1; 1s_2 \left| \frac{1}{2}S_z \right. \right\rangle \\ &= \frac{1}{\sqrt{2(p_1 \cdot \bar{P} + m_1 M_0)}} \bar{u}(p_1, \lambda_1) \Gamma u(\bar{P}, S_z), \end{aligned} \quad (2)$$

where

$$\Gamma = -\frac{1}{\sqrt{3}} \gamma_5 \not{\epsilon}(p_2, \lambda_2), \quad m_1 = m_Q, \quad m_2 = m_{[ud]}, \quad \bar{P} = p_1 + p_2, \quad (3)$$

and

$$\varphi(x, k_\perp) = A\phi, \quad (4)$$

with $\phi = 4(\frac{\pi}{\beta^2})^{3/4} \frac{e_1 e_2}{x_1 x_2 M_0} \exp(\frac{-\mathbf{k}^2}{2\beta^2})$ and $A = \sqrt{\frac{12(M_0 m_1 + p_1 \cdot \bar{P})}{12M_0 m_1 + 4p_1 \bar{P} + 8p_1 \cdot p_2 p_2 \cdot \bar{P}/m_2^2}}$ which can be obtained by normalizing the state $|\Sigma_Q(P, S, S_z)\rangle$,

$$\langle \Sigma_Q(P', S', S'_z) | \Sigma_Q(P, S, S_z) \rangle = 2(2\pi)^3 P^+ \delta^3(\tilde{P}' - \tilde{P}) \delta_{S'S} \delta_{S'_z S_z}. \quad (5)$$

All other notations can be found in Ref.[1].

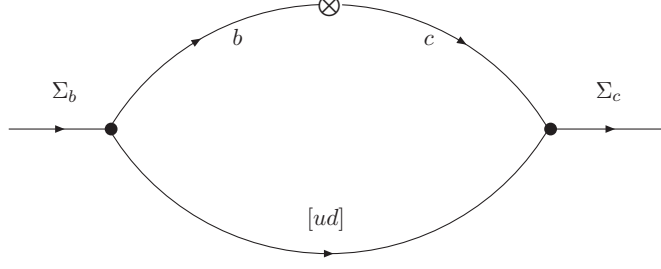


FIG. 1: Feynman diagram for $\Sigma_b \rightarrow \Sigma_c$ transitions, where \otimes denotes $V - A$ current vertex.

B. $\Sigma_b \rightarrow \Sigma_c$ transition form factors

The lowest order Feynman diagram for the $\Sigma_b \rightarrow \Sigma_c$ weak decay is shown in Fig. 1. Using the wavefunction for $|\Sigma_Q(P, S, S_z)\rangle$, we obtain

$$\begin{aligned} & \langle \Sigma_{Q'}(P', S'_z) | \bar{Q}' \gamma^\mu (1 - \gamma_5) Q | \Sigma_Q(P, S_z) \rangle \\ &= \int \{d^3 \tilde{p}_2\} \frac{\phi_{\Sigma_{Q'}}^*(x', k'_\perp) \phi_{\Sigma_Q}(x, k_\perp)}{2\sqrt{p_1^+ p_1'^+ (p_1 \cdot \bar{P} + m_1 M_0)(p_1' \cdot \bar{P}' + m_1' M_0')}} \\ & \times \bar{u}(\bar{P}', S'_z) \bar{\Gamma}'(\not{p}_1' + m_1') \gamma^\mu (1 - \gamma_5) (\not{p}_1 + m_1) \Gamma u(\bar{P}, S_z), \end{aligned} \quad (6)$$

where

$$\begin{aligned} \bar{\Gamma}' &= \gamma_0 \Gamma' \gamma_0, \\ m_1 &= m_b, \quad m_1' = m_c, \quad m_2 = m_{[ud]}, \end{aligned} \quad (7)$$

and $Q(Q')$ represent $b(c)$ quark, $p_1(p_1')$ is its momentum and $P(P')$ denotes the momentum of initial (final) baryon. From $\tilde{p}_2 = \tilde{p}_2'$, we have

$$x' = \frac{P^+}{P'^+} x, \quad k'_\perp = k_\perp + x_2 q_\perp. \quad (8)$$

with $x = x_2$, $x' = x'_2$. Thus, Eq. (6) is rewritten as

$$\begin{aligned} & \langle \Sigma_{Q'}(P', S'_z) | \bar{Q}' \gamma^\mu (1 - \gamma_5) Q | \Sigma_Q(P, S_z) \rangle \\ &= \int \frac{dx d^2 k_\perp}{2(2\pi)^3} \frac{\phi_{\Sigma_{Q'}}(x', k'_\perp) \phi_{\Sigma_Q}(x, k_\perp)}{2\sqrt{x_1 x_1' (p_1 \cdot \bar{P} + m_1 M_0)(p_1' \cdot \bar{P}' + m_1' M_0')}} \\ & \times \bar{u}(\bar{P}', S'_z) \left[\frac{1}{\sqrt{3}} \gamma_5 \varepsilon(\lambda_2')^* \right] (\not{p}_1' + m_1') \gamma^\mu (1 - \gamma_5) (\not{p}_1 + m_1) \left[\frac{1}{\sqrt{3}} \gamma_5 \varepsilon(\lambda_2) \right] u(\bar{P}, S_z). \end{aligned} \quad (9)$$

The form factors for the weak transition $\Sigma_Q \rightarrow \Sigma_{Q'}$ are defined in the standard way as

$$\begin{aligned} & \langle \Sigma_{Q'}(P', S', S'_z) | \bar{Q}' \gamma_\mu (1 - \gamma_5) Q | \Sigma_Q(P, S, S_z) \rangle \\ &= \bar{u}_{\Sigma_{Q'}}(P', S'_z) \left[\gamma_\mu f_1(q^2) + i \sigma_{\mu\nu} \frac{q^\nu}{M_{\Sigma_Q}} f_2(q^2) + \frac{q_\mu}{M_{\Sigma_Q}} f_3(q^2) \right] u_{\Sigma_Q}(P, S_z) \\ & - \bar{u}_{\Sigma_{Q'}}(P', S'_z) \left[\gamma_\mu g_1(q^2) + i \sigma_{\mu\nu} \frac{q^\nu}{M_{\Sigma_Q}} g_2(q^2) + \frac{q_\mu}{M_{\Sigma_Q}} g_3(q^2) \right] \gamma_5 u_{\Sigma_Q}(P, S_z). \end{aligned} \quad (10)$$

where $q \equiv P - P'$, Q and Q' denote b and c , respectively. Since $S = S' = 1/2$, we will be able to write $|\Sigma_Q(P, S, S'_z)\rangle$ as $|\Sigma_Q(P, S_z)\rangle$.

Following[1, 25], we extract the form factors for the weak transition matrix elements of $\Sigma_b \rightarrow \Sigma_c$ as

$$\begin{aligned}
f_1(q^2) &= \frac{1}{8P^+P'^+} \int \frac{dx_2 d^2 k_{2\perp}}{2(2\pi)^3} \frac{\phi_{00}'^*(x', k'_\perp) \phi_{00}(x, k_\perp)}{6\sqrt{x_1 x'_1 (p_1 \cdot \bar{P} + m_1 M_0)(p'_1 \cdot \bar{P}' + m'_1 M'_0)}} \\
&\quad Tr[(\bar{P} + M_0)\gamma^+(\bar{P}' + M'_0)\gamma_5\gamma_a(\not{p}'_1 + m'_1)\gamma^+(\not{p}_1 + m_1)\gamma_5\gamma_b](\frac{p_2^a p_2^b}{m_2^2} - g^{ab}) \\
g_1(q^2) &= \frac{1}{8P^+P'^+} \int \frac{dx_2 d^2 k_{2\perp}}{2(2\pi)^3} \frac{\phi_{00}'^*(x', k'_\perp) \phi_{00}(x, k_\perp)}{6\sqrt{x_1 x'_1 (p_1 \cdot \bar{P} + m_1 M_0)(p'_1 \cdot \bar{P}' + m'_1 M'_0)}} \\
&\quad Tr[(\bar{P} + M_0)\gamma^+\gamma_5(\bar{P}' + M'_0)\gamma_5\gamma_a(\not{p}'_1 + m'_1)\gamma^+\gamma_5(\not{p}_1 + m_1)\gamma_5\gamma_b](\frac{p_2^a p_2^b}{m_2^2} - g^{ab}) \\
\frac{f_2(q^2)}{M_{\Lambda_Q}} &= -\frac{1}{8P^+P'^+q_\perp^i} \int \frac{dx_2 d^2 k_{2\perp}}{2(2\pi)^3} \frac{\phi_{00}'^*(x', k'_\perp) \phi_{00}(x, k_\perp)}{6\sqrt{x_1 x'_1 (p_1 \cdot \bar{P} + m_1 M_0)(p'_1 \cdot \bar{P}' + m'_1 M'_0)}} \\
&\quad Tr[(\bar{P} + M_0)\sigma^{i+}(\bar{P}' + M'_0)\gamma_5\gamma_a(\not{p}'_1 + m'_1)\gamma^+(\not{p}_1 + m_1)\gamma_5\gamma_b](\frac{p_2^a p_2^b}{m_2^2} - g^{ab}) \\
\frac{g_2(q^2)}{M_{\Lambda_Q}} &= \frac{1}{8P^+P'^+q_\perp^i} \int \frac{dx_2 d^2 k_{2\perp}}{2(2\pi)^3} \frac{\phi_{00}'^*(x', k'_\perp) \phi_{00}(x, k_\perp)}{6\sqrt{x_1 x'_1 (p_1 \cdot \bar{P} + m_1 M_0)(p'_1 \cdot \bar{P}' + m'_1 M'_0)}} \\
&\quad Tr[(\bar{P} + M_0)\sigma^{i+}\gamma_5(\bar{P}' + M'_0)\gamma_5\gamma_a(\not{p}'_1 + m'_1)\gamma^+\gamma_5(\not{p}_1 + m_1)\gamma_5\gamma_b](\frac{p_2^a p_2^b}{m_2^2} - g^{ab}). \quad (11)
\end{aligned}$$

with $i = 1, 2$. The traces can be worked out straightforwardly and all the details can be found in Ref.[1].

C. Isgur-Wise functions of the transition

As well known under the heavy quark limit ($m_Q \rightarrow \infty$)[26], the six form factors f_i , g_i ($i=1,2,3$) are no longer independent, but are related to each other by an extra symmetry. Thus the matrix elements are determined by two universal Isgur-Wise functions $\xi_1(v \cdot v')$ and $\xi_2(v \cdot v')$.

The generalized Isgur-Wise functions in the $\Sigma_Q \rightarrow \Sigma_{Q'}$ transition are defined through the following expression

$$\begin{aligned}
&< \Sigma_{Q'}(v', S'_z) | \bar{Q}'_{v'} \gamma_\mu (1 - \gamma_5) Q_v | \Sigma_Q(v, S_z) > \\
&= \frac{1}{3} [g^{\alpha\beta} \xi_1(\omega) - v^\alpha v'^\beta \xi_2(\omega)] \bar{u}(v', S'_z) \gamma_5 (\gamma_\alpha + v'_\alpha) \gamma_\mu (1 - \gamma_5) (\gamma_\beta + v_\beta) \gamma_5 u(v, S_z), \quad (12)
\end{aligned}$$

where $\omega \equiv v \cdot v'$. In fact, as we re-calculate the transition matrix elements under the heavy quark limit, one can easily obtain a new expression corresponding to Eq.(9) where there are six independent form factors.

As discussed in Ref.[1] with a replacements in the heavy quark effective theory (HQET)

$$\begin{aligned} |\Sigma_Q(P, S_z)\rangle &\rightarrow \sqrt{M_{\Sigma_Q}} |\Sigma_Q(v, S_z)\rangle, \\ u(\bar{P}, S_z) &\rightarrow \sqrt{m_Q} u(v, S_z) \\ \phi_{\Sigma_Q}(x, k_\perp) &\rightarrow \sqrt{\frac{m_Q}{X}} \Phi(X, k_\perp), \end{aligned} \quad (13)$$

and

$$\begin{aligned} M_{\Sigma_Q} &\rightarrow m_Q, & M_0 &\rightarrow m_Q, \\ e_1 &\rightarrow m_Q, \\ e_2 &\rightarrow v \cdot p_2 = \frac{m_2^2 + k_\perp^2 + X^2}{2X}, \\ \vec{k}^2 &\rightarrow (v \cdot p_2)^2 - m_2^2, \\ \not{p}_1 + m_1 &\rightarrow m_Q(\not{v} + 1) \\ \frac{e_1 e_2}{x_1 x_2 M_0} &\rightarrow \frac{m_Q}{X} (v \cdot p_2), \end{aligned} \quad (14)$$

we are able to re-formulate the transition form factors obtained in the previous section under the heavy quark limit.

The matrix element of the transition $\Sigma_Q \rightarrow \Sigma_{Q'}$ is then

$$\begin{aligned} &\langle \Sigma_{Q'}(v', S'_z) | \bar{Q}'_{v'} \gamma^\mu (1 - \gamma_5) Q_v | \Sigma_Q(v, S_z) \rangle \\ &= - \int \frac{dX}{X} \frac{d^2 k_\perp}{2(2\pi)^3} \Phi(X, k_\perp) \Phi(X', k'_\perp) \frac{1}{3} \bar{u}(v', S'_z) \gamma_5 (\gamma_\alpha + v'_\alpha) \gamma^\mu (1 - \gamma_5) \\ &\quad (\gamma_\beta + v'_\beta) \gamma_5 u(v, S_z) \left(\frac{p_2^\alpha p_2^\beta}{m_2^2} - g^{\alpha\beta} \right), \end{aligned} \quad (15)$$

with

$$\Phi(X, k_\perp) = \sqrt{\frac{24}{16 + 8v \cdot p_2^2/m_2^2}} 4\sqrt{v \cdot p_2} \left(\frac{\pi}{\beta_\infty^2} \right)^{\frac{3}{4}} \exp \left(-\frac{(v \cdot p_2)^2 - m_2^2}{2\beta_\infty^2} \right), \quad (16)$$

where β_∞ denotes the value of β in the heavy quark limit.

Thus we can write down the transition matrix element as

$$\begin{aligned} &< \Sigma_{Q'}(v', S'_z) | \bar{Q}'_{v'} \gamma^\mu (1 - \gamma_5) Q_v | \Sigma_Q(v, S_z) > \\ &= - \int \frac{dX}{X} \frac{d^2 k_\perp}{2(2\pi)^3} \Phi(X, k_\perp) \Phi(X', k'_\perp) \frac{1}{3} \bar{u}(v', S'_z) \gamma_5 (\gamma_\alpha + v'_\alpha) \gamma^\mu (1 - \gamma_5) \\ &\quad (\gamma_\beta + v'_\beta) \gamma_5 u(v, S_z) (a_1 g^{\alpha\beta} + a_2 v^\alpha v'^\beta + a_3 v'^\alpha v^\beta + a_4 v'^\alpha v'^\beta + a_5 v^\alpha v^\beta). \end{aligned} \quad (17)$$

By the relation $\bar{u}' \gamma_5 (\not{v}' + 1) = (\not{v} + 1) \gamma_5 u = 0$, the terms with a_3 , a_4 and a_5 do not contribute to the transition, thus

$$\begin{aligned} &< \Sigma_{Q'}(v', S'_z) | \bar{Q}'_{v'} \gamma^\mu (1 - \gamma_5) Q_v | \Sigma_Q(v, S_z) > \\ &= - \int \frac{dX}{X} \frac{d^2 k_\perp}{2(2\pi)^3} \Phi(X, k_\perp) \Phi(X', k'_\perp) \frac{1}{3} \bar{u}(v', S'_z) \gamma_5 (\gamma_\alpha + v'_\alpha) \gamma^\mu (1 - \gamma_5) \\ &\quad (\gamma_\beta + v'_\beta) \gamma_5 u(v, S_z) (a_1 g^{\alpha\beta} + a_2 v^\alpha v'^\beta), \end{aligned} \quad (18)$$

TABLE I: Quark mass and the parameter β (in units of GeV).

m_c	m_b	$m_{[ud]}$	$\beta_{c[ud]}$	$\beta_{b[ud]}$
1.3	4.4	0.77	0.45	0.50

and

$$\begin{aligned}
 a_1 &= -\frac{(w^2 - 1)p_2^2 + 2v \cdot p_2 v' \cdot p_2 \omega - (v' \cdot p_2)^2 - (v \cdot p_2)^2}{2m_2^2(\omega^2 - 1)} \\
 a_2 &= -\frac{\omega(\omega^2 - 1)p_2^2 - 2v \cdot p_2 v' \cdot p_2(2\omega^2 + 1) + 3\omega[(v' \cdot p_2)^2 + (v \cdot p_2)^2]}{2m_2^2(\omega^2 - 1)^2}
 \end{aligned} \tag{19}$$

Comparing Eq.(22) with Eq. (12), we get

$$\xi_1 = - \int \frac{dX}{X} \frac{d^2 k_\perp}{2(2\pi)^3} \Phi(X, k_\perp) \Phi(X', k'_\perp) a_1, \tag{20}$$

$$\xi_2 = \int \frac{dX}{X} \frac{d^2 k_\perp}{2(2\pi)^3} \Phi(X, k_\perp) \Phi(X', k'_\perp) a_2. \tag{21}$$

The forms of ξ_1 and ξ_2 are similar to that in Eq.(4.18) and Eq. (4.19) of Ref.[25] and can be directly evaluated in the time-like region by choosing a reference frame where $q_\perp = 0$.

III. NUMERICAL RESULTS

In this section we present our numerical results for the transition $\Sigma_b \rightarrow \Sigma_c$ along with all input parameters. First we need to obtain the form factors, then using them the predictions on semi-leptonic processes $\Sigma_b \rightarrow \Sigma_c l \bar{\nu}_l$ and non-leptonic decays $\Sigma_b \rightarrow \Sigma_c M^-$ (M represents π , K , ρ , K^* , a_1 etc.) will be made.

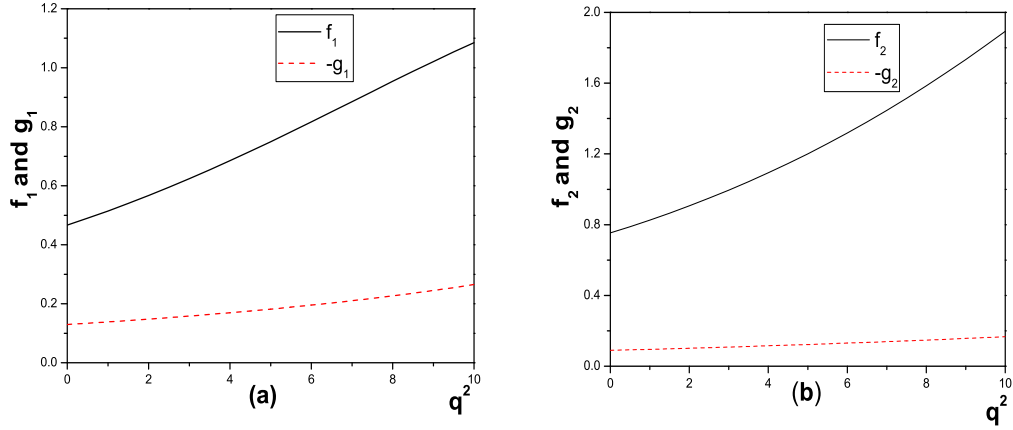
First of all, let us list our input parameters. The baryon masses $M_{\Sigma_b} = 5.807$ GeV, $M_{\Sigma_c} = 2.452$ GeV are taken from[27]. For the heavy quark masses, we set m_b and m_c following Ref.[19]. In the early literature, the mass of the constituent light axial vector diquark $m_{[ud]}$ disperses in a rather wide range, for example, it is set as: 614-618 MeV[28], 770 MeV[6], 909 MeV[7]. In [1] we fixed the scalar diquark mass as $m_{[ud]_S} = 500$ MeV. Generally an axial vector should be slightly heavier than a scalar with the same constituents, so we set $m_{[ud]_V} = 770$ MeV. Since the $[ud]$ diquark mass is close to the mass of a strange quark, we may assume that the parameters $\beta_{b[ud]}$ and $\beta_{c[ud]}$ should be close to $\beta_{b\bar{s}}$ and $\beta_{c\bar{s}}$ which appear in the meson case[19]. All the input parameters are collected in Table I.

A. $\Sigma_b \rightarrow \Sigma_c$ form factors and the Isgur-Wise functions

As discussed in Ref.[19] the form factors are calculated in the frame $q^+ = 0$ with $q^2 = -q_\perp^2 \leq 0$ (the space-like region). To extended them into the time-like region, an analytic

TABLE II: The $\Sigma_b \rightarrow \Sigma_c$ form factors given in the three-parameter form.

F	$F(0)$	a	b
f_1	0.4664	2.32	3.40
f_2	0.7358	2.08	2.08
g_1	-0.1298	1.15	0.42
g_2	-0.08977	1.11	1.07


 FIG. 2: (a) Form factors f_1 and g_1 (b) Form factors f_2 and g_2

three-parameter form was suggested [25]

$$F(q^2) = \frac{F(0)}{\left(1 - \frac{q^2}{M_{\Sigma_b}^2}\right) \left[1 - a \left(\frac{q^2}{M_{\Sigma_b}^2}\right) + b \left(\frac{q^2}{M_{\Sigma_b}^2}\right)^2\right]}, \quad (22)$$

where $F(q^2)$ stands for the form factors $f_{1,2}$ and $g_{1,2}$. a , b and $F(0)$ in $F(q^2)$ are parameters which need to be fixed using the form factors in the space-like region we calculate numerically. This form can be automatically extended into the time-like, i.e. physical region with $q^2 \geq 0$. The fitted values of a , b and $F(0)$ in the form factors $f_{1,2}$ and $g_{1,2}$ are presented in Table II. The dependence of the form factors on q^2 is depicted in Fig. 2.

The values shown in Table II and Fig. 2 indicate that the form factor g_1 and g_2 are small compared with f_1 and f_2 and $f_1(f_2)$ and $g_1(g_2)$ have opposite signs, this is similar to the

case of $\Theta_b \rightarrow \Theta_c$ [25].

Now let us turn to re-calculate the transition amplitude in the HQET. In the heavy quark limit, we choose $\beta^\infty = 0.50$ GeV for Σ_b and Σ_c . The Isgur-Wise function is parameterized as

$$\xi(\omega) = 1 - \rho^2(\omega - 1) + \frac{\sigma^2}{2}(\omega - 1)^2 + \dots, \quad (23)$$

where $\rho^2 \equiv -\frac{d\xi(\omega)}{d\omega}|_{\omega=1}$ is the slope parameter and $\sigma^2 \equiv \frac{d^2\xi(\omega)}{d\omega^2}|_{\omega=1}$ is the curvature of the Isgur-Wise function. Our fitted values are

$$\xi_1 = 1 - 2.09(\omega - 1) + 1.84(\omega - 1)^2 \quad (24)$$

$$\xi_2 = 0.42[1 - 2.79(\omega - 1) + 3.09(\omega - 1)^2]. \quad (25)$$

The Isgur-Wise functions in the whole ω range is depicted in Fig. 3. One can notice that $\xi_1(\omega = 1) = 1$ holds as required by the normalization of the Isgur-Wise function. Even though, as indicated in literature, $\xi_2(\omega = 1)$ is unknown, at the large N_C limit it is determined to be $1/2$ [29] and other early model-dependent studies also confirm this prediction[30].

It is worth indicating clearly that under the heavy quark limit, i.e. $M_Q \rightarrow \infty$, the mass of heavy quark disappears in the wavefunction (20), but the light constituent mass (anti-quark for meson case and diquark for baryon case) remains. Therefore the theoretical evaluation on the transition rate weakly depends on the light constituent mass even under the heavy quark limit.

From Fig. 3, we observe that $\xi_2|_{\omega=1} = 0.42$ which is slightly lower than $1/2$. This deviation is due to the mass $m_{diquark}$ in the assumed wavefunction (see Eq.(21)). To further explore the dependence, we deliberately vary $m_{[ud]_V}$, $\beta_{b[ud]}$ and $\beta_{c[ud]}$. We find that ξ_1 does not change at all, but the intercept $\xi_2(\omega = 1)$ changes for different values of $m_{[ud]_V}$, $\beta_{b[ud]}$ and $\beta_{c[ud]}$. For example, as $\xi_1 = 1$ and $\xi_2 = 0.47$ when one sets $m_{[ud]_V} = 0.5$ GeV, $\beta_{b[ud]} = 0.4$ and $\beta_{c[ud]} = 0.35$. Definitely non-zero $m_{diquark}$ breaks the heavy quark symmetry $SU_f(2) \otimes SU_s(2)$, but the violation is still rather small, so that one can use the simplified expression with only two Isgur-Wise functions to approach the transition matrix elements.

B. Semi-leptonic decay of $\Sigma_b \rightarrow \Sigma_c + l\bar{\nu}_l$

With the form factors given in last subsection, we are able to calculate the width of $\Sigma_b \rightarrow \Sigma_c l\bar{\nu}_l$.

In table III we list our numerical results. The predictions are presented for two cases: with and without taking the heavy quark limit.

It is also interesting to study the longitudinal and transverse helicity amplitudes $H_{\lambda', \lambda_W}^{V,A}$ where λ' and λ_W are the helicities of the daughter baryon and the emitted W-boson respectively, since it may provide more information about the model and even the whole framework. Moreover, several asymmetry parameters a_L , a_T and P_L are defined in earlier literature and in this work for readers' convenience we explicitly present them in the appendix. A ratio of

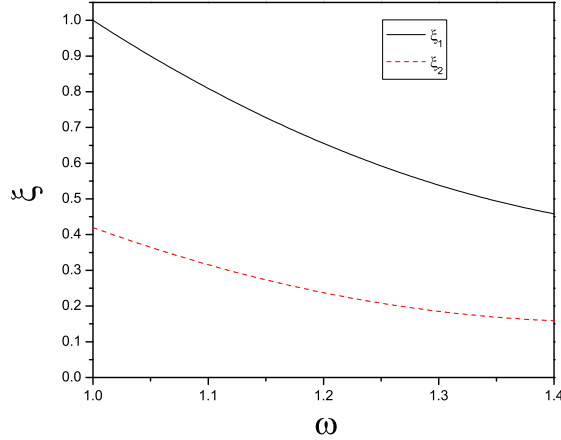


FIG. 3: The $\Sigma_b \rightarrow \Sigma_c$ Isgur-Wise function $\zeta(\omega)$ with diquark mass $m_{[ud]_V} = 770$ MeV.

TABLE III: The widths (in unit 10^{10}s^{-1}) and polarization asymmetries of $\Sigma_b \rightarrow \Sigma_c l \bar{\nu}_l$.

	width	a_L	a_T	Γ_L	Γ_T	R	P_L
this work ^a	1.38	0.715	-0.893	1.06	0.32	3.25	0.337
this work ^b	1.60	0.706	-0.966	1.09	0.51	2.13	0.171
spectator-quark model ^a [8]	4.3	-	-	3.93	0.37	10.7	-
relativistic quark model ^b [7]	1.44	-	-	1.23	0.21	5.89	-
the Bethe-Salpeter approach ^b [10]	1.65	-	-	-	-	-	-
relativistic three-quark model ^b [9]	2.23	-	-	1.90	0.33	5.76	-

^a without the heavy quark limit

^bwith the heavy quark limit

longitudinal to transverse rates R is also defined (see the appendix too), and $R > 1$ implies that the longitudinal polarization dominates. Because the values of such asymmetries are more sensitive to the details of the employed models, comparing the theoretical predictions on them with the data which will be available soon at LHC as expected, can help to gain a better understanding of the models.

In Tab.III the predictions achieved with other approaches [7] are also presented. One notices from Tab.III that there is an obvious discrepancy between predictions on the semi-leptonic decay widths and R values estimated by different models. The future experimental measurements would provide a chance to test the applicability of different approaches.

TABLE IV: widths (in unit 10^{10}s^{-1}) and up-down asymmetries of non-leptonic decays $\Sigma_b \rightarrow \Sigma_c M$ with the light diquark mass $m_{[ud]} = 500$ MeV.

	without the heavy quark limit		with the heavy quark limit	
	width	α	width	α
$\Sigma_b^0 \rightarrow \Sigma_c^+ \pi^-$	0.140	0.514	0.146	0.556
$\Sigma_b^0 \rightarrow \Sigma_c^+ \rho^-$	0.166	-0.653	0.0907	-0.785
$\Sigma_b^0 \rightarrow \Sigma_c^+ K^-$	0.0115	0.510	0.0118	0.548
$\Sigma_b^0 \rightarrow \Sigma_c^+ K^{*-}$	0.00864	-0.629	0.00471	-0.750
$\Sigma_b^0 \rightarrow \Sigma_c^+ a_1^-$	0.163	-0.551	0.0880	-0.646
$\Sigma_b^0 \rightarrow \Sigma_c^+ D_s^-$	0.796	0.379	0.655	0.425
$\Sigma_b^0 \rightarrow \Sigma_c^+ D_s^{*-}$	0.292	-0.302	0.152	-0.317
$\Sigma_b^0 \rightarrow \Sigma_c^+ D^-$	0.0266	0.408	0.0242	0.440
$\Sigma_b^0 \rightarrow \Sigma_c^+ D^{*-}$	0.0137	-0.331	0.00694	-0.356

C. Non-leptonic decays of $\Sigma_b \rightarrow \Sigma_c + M$

From the theoretical aspects, calculating the concerned quantities of the non-leptonic decays seem to be much more complicated than the semi-leptonic ones. Our theoretical framework is based on the factorization assumption, namely the hadronic transition matrix element is factorized into a product of two independent matrix elements of currents. One of them is determined by a decay constant whereas the other is decomposed into a sum of a few terms according to the Lorentz structure of the current and their coefficients are the to-be-determined form factors. The decays $\Sigma_b^0 \rightarrow \Sigma_c + M^-$ is the so-called color-favored transition, thus and factorization should be a good approximation. Therefore, the study on these non-leptonic decays can be a check of the consistency of the obtained form factors in the heavy bottomed baryon system.

The formulas of the decay rates for non-leptonic decays $\Sigma_b \rightarrow \Sigma_c + M$ in the factorization approach are given in Ref.[31] and collected in our previous paper [1]. Our numerical results are shown in Tab.IV. The CKM matrix elements, the effective Wilson coefficient $a_1 = 1$ and the meson decay constants are the same as in Ref.[1].

IV. Two comments are made:

- (1) The ratio $\frac{BR(\Sigma_b^0 \rightarrow \Sigma_c^+ l^- \bar{\nu}_l)}{BR(\Sigma_b^0 \rightarrow \Sigma_c^+ \pi^-)}$ is 11.4 which will be experimentally tested.
- (2) The up-down asymmetry α for $\Sigma_b \rightarrow \Sigma_c V$ is negative but that for $\Sigma_b \rightarrow \Sigma_c P$ is positive where α is defined in the appendix .

D. Estimate on the transition of $\Omega_b \rightarrow \Omega_c$

Though we focus on the transition of $\Sigma_b \rightarrow \Sigma_c$ in this work, the formulas deduced in section II can be applied to calculate the transition between the $\frac{1}{2}^+$ baryons such as Ω_b

TABLE V: Various theoretical predictions on the rates $\Omega_b \rightarrow \Omega_c e \nu$ and $\Sigma_b \rightarrow \Sigma_c e \nu$ (in unit 10^{10}s^{-1})

Decay	[7]	[8]	[9]	[10]
$\Sigma_b \rightarrow \Sigma_c e \nu$	1.44	4.3	2.23	1.65
$\Omega_b \rightarrow \Omega_c e \nu$	1.29	5.4	1.87	1.81

and Ω_c whose structure is analogous to $\Sigma_{b(c)}$ in the quark-diquark picture i.e. the diquark is an axial vector. Since the light diquark is regarded as a spectator, under the SU(3) symmetry of light quarks the predictions on the decay rates of $\Sigma_b \rightarrow \Sigma_c$ hold for $\Omega_b \rightarrow \Omega_c$ approximatively. As Ω_b decays via only weak interaction the branching ratios of $\Omega_b \rightarrow \Omega_c$ should be dominant, so that these decays can be detected more easily. Undoubtedly, since the SU(3) symmetry is slightly broken, different input parameters would bring up minor differences for the numerical results of $\Omega_b \rightarrow \Omega_c$ from $\Sigma_b \rightarrow \Sigma_c$, but the deviation should be relatively small, and the allegation is supported by some theoretical studies which compare the width of $\Omega_b \rightarrow \Omega_c e \nu$ with $\Sigma_b \rightarrow \Sigma_c e \nu$ (Tab. V).

IV. CONCLUSIONS AND DISCUSSIONS

In this paper, we extensively explore the $\Sigma_b \rightarrow \Sigma_c$ transition in all details and estimate the widths for the semi-leptonic decay and non-leptonic two-body decays of $\Sigma_b \rightarrow \Sigma_c$ as well as several relevant measurable quantities. For the heavy baryons the quark-diquark picture is employed, which reduces the three-body structure into a two-body one.

The matrix elements of the transition $\Sigma_b \rightarrow \Sigma_c$ can be parameterized with a few form factors f_i and g_i ($i = 1, 2, 3$) according to the Lorentz structures, and we obtain these form factors by calculating the transition $\Sigma_b \rightarrow \Sigma_c$ in the LFQM and evaluate them numerically. The form factors f_1 and f_2 for $\Sigma_b \rightarrow \Sigma_c$ are much larger than g_1 and g_2 , it is noted that $f_1(f_2)$ and $g_1(g_2)$ have opposite signs. Furthermore, we also derive the generalized Isgur-Wise functions ξ_1 and ξ_2 under the heavy quark limit. We find that $\xi_1(\omega = 1) = 1$ is consistent with the normalization condition, but $\xi_2(\omega = 1)$ is slightly lower than $1/2$ which was predicted by large N_c theory. Our analysis indicates that the deviation is due to the non-zero mass of the light constituents in hadrons (meson and baryon). With the form factors derived in terms of the LFQM or the Isgur-Wise functions we evaluate the semi-leptonic decay rates of $\Sigma_b \rightarrow \Sigma_c$ with and without taking the heavy quark limit. The results with and without heavy quark limit do not decline much from each other, moreover, our numerical results of the rates are generally consistent with that estimated by different approaches. However, it is interesting to note that for the transverse polarization asymmetry P_L , there is an obvious discrepancy between our results and those by other approaches. Moreover, in terms of SU(3) symmetry of light quarks we estimate the rates $\Omega_b \rightarrow \Omega_c$ which is approximately equal to those of $\Sigma_b \rightarrow \Sigma_c$.

Since the LHCb is running successfully and a remarkable amount of data on Σ_b and Ω_b production and decay is being accumulated, especially by the LHCb detector, thus we have

all confidence that in near future (maybe not next year, but anyhow won't be too far away), their decay rates and even the asymmetries would be more accurately measured, and we will have a great opportunity to testify our models.

Now let us estimate the feasibility of observing the decay process $\Sigma_b \rightarrow \Sigma_c l \nu_l$. Firstly, we use the code PYTHIA8.1 to calculate the production cross section of Σ_b^+ via $pp \rightarrow b\bar{b} \rightarrow \Sigma_b^+ + X$. By the PYTHIA8.1[32–34], 100000 $b\bar{b}$ pairs are generated at the $E_{\text{CM}} = 7\text{TeV}$. Then $N_{\Sigma_b^+} = 354$ are produced and the corresponding production cross section is $\sigma_{\Sigma_b^+} \approx 354/100000 = 3.54 \times 10^{-3} \sigma_{b\bar{b}}$. In 2011, the integrated luminosity of the LHCb is 1 fb^{-1} [35] and the production cross section of the $b\bar{b}$ pairs is $\sigma_{b\bar{b}} = 288 \mu\text{b}$ [36], so our estimate is that about $1.02 \times 10^9 \Sigma_b^+$ exist in the 2011 data.

Because the LHCb detector is good at tracking the charged particles, such as p^\pm , K^\pm and π^\pm and charged leptons, we suggest the decay chain used to find the Σ_b 's semileptonic decay is that: $\Sigma_b^+ \rightarrow \Sigma_c^0 \mu^+ \nu_\mu$, $\Sigma_c^0 \rightarrow \Lambda_c^+ \pi^-$ (the branching ratio is $\Gamma_{\Lambda_c^+ \pi^-} / \Gamma_{\Sigma_c} \approx 100\%$) and $\Lambda_c^+ \rightarrow p K^+ \pi^-$ (the branching ratio is $\Gamma_{p K^+ \pi^-} / \Gamma_{\Lambda_c} \approx 5\%$). So, the measurable number of this decay chain is:

$$\mathcal{N}_{\Sigma_b} = 1.02 \times 10^9 \times \frac{\Gamma_{\Sigma_c \mu \nu_\mu}}{\Gamma_{\Sigma_b}} \times 100\% \times 5\% \times \epsilon_{\text{trig}} \times \epsilon_\theta, \quad (26)$$

where, $\Gamma_{\Sigma_c \mu \nu_\mu} / \Gamma_{\Sigma_b}$ stands for the branching ratio of Σ_b 's semi-leptonic decay, the ϵ_{trig} is the efficiency of the detection trigger and $\epsilon_\theta \approx 20\%$ is the efficiency of the detector's geometric acceptance[37]. Without losing generality, we set $\epsilon_{\text{trig}} = 88\%$ here (also given in [37]). The decay width of Σ_b is not known yet. Since QCD is flavor-blind, thus we have reason to believe that considering the phase space of final state the decay width of the Σ_b should be related to that of Γ_{Λ_b} , Γ_{Λ_c} and Γ_{Σ_c} as:

$$\Gamma_{\Sigma_b} = \Gamma_{\Lambda_b} \frac{\Gamma_{\Sigma_c}}{\Gamma_{\Lambda_c}} = 4.81 \times 10^{10} (\text{in unit } 10^{10} \text{s}^{-1}). \quad (27)$$

Substituting all these values back into Eq.(26) ($\Gamma_{\Sigma_c \mu \nu_\mu} = 1.38 \times 10^{10} \text{s}^{-1}$ can be found in Tab-III), we have the number of signals of $\Sigma_b \rightarrow \Sigma_c + l \bar{\nu}$:

$$\mathcal{N}_{\Sigma_b} = 1.87 \times 10^{-4} \frac{\Gamma_{\Sigma_c \mu \nu_\mu}}{10^{10} \text{s}^{-1}} = 2.58 \times 10^{-4}. \quad (28)$$

If the luminosity of LHCb is not increased greatly in the future to avoid the high level pile-up, we conclude that, since the strong decay $\Sigma_b \rightarrow \Lambda_b \pi$ dominates and the lifetime of Σ_b is determined by the mode, the branching ratio of the weak decay is significantly suppressed, it would be hard to directly observe the signals of semileptonic decays of Σ_b . Since the signal of the semileptonic decays of Σ_b is clear and related to new physics, so that is worth careful investigation at LHCb, even though it is almost impossible to be measured for the present luminosity if only SM applies. Thus, as analyzed above, we would recommend to measure $\Omega_b \rightarrow \Omega_c$ transitions because Ω_b does not decay via strong interaction, so $\Omega_b \rightarrow \Omega_c + l \bar{\nu}$ and $\Omega_b \rightarrow \Omega_c + M$ would be the dominant modes. It enables us to make a more precise measurement by which we can not only further investigate the validity of the diquark picture

for heavy baryons, but also create an opportunity to search for new physics beyond the SM, at least check if the new physics scenario shows up in such transitions. We also suggest to measure the quantities such as the asymmetries besides the widths, because of the obvious advantages about our models and physics.

Acknowledgement

This work is supported by the National Natural Science Foundation of China (NNSFC) under the contract No. 11075079, No. 11175091 and No. 11005079; the Special Grant for the Ph.D. program of Ministry of Education of P.R. China No. 20100032120065.

Appendix A: Semi-leptonic decays of $\Sigma_b \rightarrow \Sigma_c l \bar{\nu}_l$

The helicity amplitudes are related to the form factors for $\Sigma_b \rightarrow \Sigma_c$ through the following expressions [38]

$$\begin{aligned} H_{\frac{1}{2},0}^V &= \frac{\sqrt{Q_-}}{\sqrt{q^2}} \left((M_{\Sigma_b} + M_{\Sigma_c}) f_1 - \frac{q^2}{M_{\Sigma_b}} f_2 \right), \\ H_{\frac{1}{2},1}^V &= \sqrt{2Q_-} \left(-f_1 + \frac{M_{\Sigma_b} + M_{\Sigma_c}}{M_{\Sigma_b}} f_2 \right), \\ H_{\frac{1}{2},0}^A &= \frac{\sqrt{Q_+}}{\sqrt{q^2}} \left((M_{\Sigma_b} - M_{\Sigma_c}) g_1 + \frac{q^2}{M_{\Sigma_b}} g_2 \right), \\ H_{\frac{1}{2},1}^A &= \sqrt{2Q_+} \left(-g_1 - \frac{M_{\Sigma_b} - M_{\Sigma_c}}{M_{\Sigma_b}} g_2 \right). \end{aligned} \quad (A1)$$

where $Q_{\pm} = 2(P \cdot P' \pm M_{\Sigma_b} M_{\Sigma_c})$. The amplitudes for the negative helicities are obtained in terms of the relation

$$H_{-\lambda' - \lambda_W}^{V,A} = \pm H_{\lambda', \lambda_W}^{V,A}, \quad (A2)$$

where the upper (lower) sign corresponds to V(A). The helicity amplitudes are

$$H_{\lambda', \lambda_W} = H_{\lambda', \lambda_W}^V - H_{\lambda', \lambda_W}^A. \quad (A3)$$

The helicities of the W -boson λ_W can be either 0 or 1, which correspond to the longitudinal and transverse polarizations, respectively. The longitudinally (L) and transversely (T) polarized rates are respectively[38]

$$\begin{aligned} \frac{d\Gamma_L}{d\omega} &= \frac{G_F^2 |V_{cb}|^2}{(2\pi)^3} \frac{q^2 p_c M_{\Sigma_c}}{12 M_{\Sigma_b}} \left[|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right], \\ \frac{d\Gamma_T}{d\omega} &= \frac{G_F^2 |V_{cb}|^2}{(2\pi)^3} \frac{q^2 p_c M_{\Sigma_c}}{12 M_{\Sigma_b}} \left[|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]. \end{aligned} \quad (A4)$$

where p_c is the momentum of Σ_c in the rest frame of Σ_b .

The integrated longitudinal and transverse asymmetries defined as

$$\begin{aligned} a_L &= \frac{\int_1^{\omega_{\max}} d\omega \, q^2 \, p_c \left[|H_{\frac{1}{2},0}|^2 - |H_{-\frac{1}{2},0}|^2 \right]}{\int_1^{\omega_{\max}} d\omega \, q^2 \, p_c \left[|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right]}, \\ a_T &= \frac{\int_1^{\omega_{\max}} d\omega \, q^2 \, p_c \left[|H_{\frac{1}{2},1}|^2 - |H_{-\frac{1}{2},-1}|^2 \right]}{\int_1^{\omega_{\max}} d\omega \, q^2 \, p_c \left[|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]}. \end{aligned} \quad (\text{A5})$$

The ratio of the longitudinal to transverse decay rates R is defined by

$$R = \frac{\Gamma_L}{\Gamma_T} = \frac{\int_1^{\omega_{\max}} d\omega \, q^2 \, p_c \left[|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 \right]}{\int_1^{\omega_{\max}} d\omega \, q^2 \, p_c \left[|H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]}, \quad (\text{A6})$$

and the longitudinal Σ_c polarization asymmetry P_L is given as

$$P_L = \frac{\int_1^{\omega_{\max}} d\omega \, q^2 \, p_c \left[|H_{\frac{1}{2},0}|^2 - |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},1}|^2 - |H_{-\frac{1}{2},-1}|^2 \right]}{\int_1^{\omega_{\max}} d\omega \, q^2 \, p_c \left[|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{\frac{1}{2},1}|^2 + |H_{-\frac{1}{2},-1}|^2 \right]}. \quad (\text{A7})$$

-
- [1] H. W. Ke, X. Q. Li and Z. T. Wei, Phys. Rev. D **77**, 014020 (2008) [arXiv:0710.1927 [hep-ph]].
 - [2] H. Park *et al.* [HyperCP Collaboration], Phys. Rev. Lett. **94**, 021801 (2005) [arXiv:hep-ex/0501014].
 - [3] X. G. He, J. Tandean and G. Valencia, Phys. Rev. Lett. **98**, 081802 (2007) [arXiv:hep-ph/0610362].
 - [4] X. G. He, J. Tandean and G. Valencia, Phys. Lett. B **631**, 100 (2005) [arXiv:hep-ph/0509041].
 - [5] N. Isgur and M. Wise, Nucl. Phys. B **348**, 276 (1991); H. Georgi, Nucl. Phys. B **348**, 293 (1991).
 - [6] J. G. Korner and P. Kroll, Z. Phys. C **57**, 383 (1993).
 - [7] D. Ebert, R. N. Faustov and V. O. Galkin, Phys. Rev. D **73**, 094002 (2006) [arXiv:hep-ph/0604017].
 - [8] R. L. Singleton, Phys. Rev. D **43**, 2939 (1991).
 - [9] M. A. Ivanov, V. E. Lyubovitskij, J. G. Korner and P. Kroll, Phys. Rev. D **56**, 348 (1997) [arXiv:hep-ph/9612463].
 - [10] M. A. Ivanov, J. G. Korner, V. E. Lyubovitskij and A. G. Rusetsky, Phys. Rev. D **59**, 074016 (1999) [arXiv:hep-ph/9809254].
 - [11] F. Wilczek, arXiv: hep-ph/0409168.
 - [12] P. Guo, H. Ke, X. Li, C. Lu and Y. Wang, Phys. Rev. D **75**, 054017 (2007).
 - [13] Y. Yu, H. Ke, Y. Ding, X. Guo, H. Jin, X. Li, P. Shen and G. Wang, Commun. Theor. Phys. **46**, 1031 (2006); Y. Yu, H. Ke, Y. Ding, X. Guo, H. Jin, X. Li, P. Shen and G. Wang, arXiv: hep-ph/0611160.
 - [14] Z. T. Wei, H. W. Ke and X. Q. Li, Phys. Rev. D **80**, 094016 (2009) [arXiv:0909.0100 [hep-ph]].

- [15] Y. Dai, X. Guo and C. Huang, Nucl.Phys. **B412** (1994) 277.
- [16] W. Jaus, Phys. Rev. D **41**, 3394 (1990); D **44**, 2851 (1991); W. Jaus, Phys. Rev. D **60**, 054026 (1999).
- [17] C. R. Ji, P. L. Chung and S. R. Cotanch, Phys. Rev. D **45**, 4214 (1992).
- [18] H. Y. Cheng, C. Y. Cheung and C. W. Hwang, Phys. Rev. D **55**, 1559 (1997) [arXiv:hep-ph/9607332].
- [19] H. Y. Cheng, C. K. Chua and C. W. Hwang, Phys. Rev. D **69**, 074025 (2004).
- [20] C. W. Hwang and Z. T. Wei, J. Phys. G **34**, 687 (2007); C. D. Lu, W. Wang and Z. T. Wei, Phys. Rev. D **76**, 014013 (2007) [arXiv:hep-ph/0701265].
- [21] H. M. Choi, Phys. Rev. D **75**, 073016 (2007) [arXiv:hep-ph/0701263];
- [22] H. W. Ke, X. Q. Li and Z. T. Wei, Phys. Rev. D **80**, 074030 (2009) [arXiv:0907.5465 [hep-ph]]; H. W. Ke, X. Q. Li, Z. T. Wei and X. Liu, Phys. Rev. D **82**, 034023 (2010) [arXiv:1006.1091 [hep-ph]].
- [23] G. Li, F. I. Shao and W. Wang, Phys. Rev. D **82**, 094031 (2010) [arXiv:1008.3696 [hep-ph]].
- [24] Z. T. Wei, H. W. Ke and X. F. Yang, Phys. Rev. D **80**, 015022 (2009) [arXiv:0905.3069 [hep-ph]]; H. W. Ke, X. Q. Li and Z. T. Wei, Eur. Phys. J. C **69**, 133 (2010) [arXiv:0912.4094 [hep-ph]]; H. W. Ke, X. H. Yuan and X. Q. Li, Int. J. Mod. Phys. A **26**, 4731 (2010), arXiv:1101.3407 [hep-ph]. H. W. Ke and X. Q. Li, Eur. Phys. J. C **71**, 1776 (2011) [arXiv:1104.3996 [hep-ph]]; H. W. Ke and X. Q. Li, Phys. Rev. D **84**, 114026 (2011) [arXiv:1107.0443 [hep-ph]]; H. W. Ke and X. Q. Li, Eur. Phys. J. C **71**, 1776 (2011) [arXiv:1104.3996 [hep-ph]].
- [25] H. Cheng and C. Chua, Phys. Rev. D **70**, 034007 (2004).
- [26] For a review, see M. Neubert, Phys. Rept. **245**, 259 (1994).
- [27] K. Nakamura *et al.* [Particle Data Group], J. Phys. G **37**, 075021 (2010).
- [28] B. Ram and V. Kriss, Phys. Rev. D **35**, 400 (1987).
- [29] C. K. Chow, Phys. Rev. D **51**, 1224 (1995) [arXiv:hep-ph/9408364].
- [30] H. Y. Cheng, Phys. Rev. D **56**, 2799 (1997) [arXiv:hep-ph/9612223].
- [31] J. Körner and M. Krämer, Z. Phys. C **55**, 659 (1992).
- [32] M. Bargiotti and V. Vagnoni, CERN-LHCB-2007-042.
- [33] N. Brambilla *et al.*, Eur. Phys. J. C **71**, 1534 (2011) [arXiv:1010.5827 [hep-ph]].
- [34] T. Sjostrand, S. Mrenna and P. Z. Skands, Comput. Phys. Commun. **178**, 852 (2008) [arXiv:0710.3820 [hep-ph]].
- [35] E. Rodrigues [LHCb Collaboration], J. Phys. Conf. Ser. **335**, 012034 (2011).
- [36] R. Aaij *et al.* [LHCb Collaboration], Eur. Phys. J. C **71**, 1645 (2011) [arXiv:1103.0423 [hep-ex]].
- [37] J. He and F. t. L. collaboration, arXiv:1001.5370 [hep-ex].
- [38] J. Körner and M. Krämer, Phys. Lett. B **275**, 495 (1992); P. Bialas, J. Körner, M. Krämer, and K. Zalewski, Z. Phys. C **57**, 115 (1993); J. Körner, M. Krämer and D. Pirjol, Prog. Part. Nucl. Phys. **33**, 787 (1994).